SAG 2006

DEPARTMENT OF MINING ENGINEERING
UNIVERSITY OF BRITISH COLUMBIA
Vancouver, B. C., Canada

COMPARATIVE EVALUATION OF GRINDING MEDIA CONSUMPTION RATES AT FULL PLANT SCALE

By Jaime Sepulveda[1], Bernie De Araugo[2] and Leigh Wardell-Johnson[3]

[1]Vice-President Moly Cop Grinding Systems, South American Operations, Santiago Chile; [2]Marketing Manager/Director, Donhad Pty Ltd, Perth Australia; [3]Marketing Manager – WA, Donhad Pty Ltd, Perth Australia.

ABSTRACT

Due recognition of the significant impact of the cost of grinding media on the overall economics of comminution facilities worldwide has created the need for reliable, practical methodologies to compare – over extended control periods - different operating conditions, arising from eventual changes in ore type, ball supplier or simply, a trial of new products from the same supplier. The current publication describes the main aspects to be considered in the planning and execution of a full scale evaluation campaign, the actual data to be recorded and the required calculation routines, including the theoretical framework justifying their applicability.

The Linear Wear Model, here described, provides a theoretical framework for the best estimations of comparative grinding media wear performance (in the absence of ball breakage) in any given application, on the basis of the Specific Wear Rate Constant, k_d^E (μm/(kWh/ton)), derived from the Specific Consumption Rate (gr/kWh), corrected by actual make-up ball size (mm).

Ideally, evaluations should be conducted in parallel grinding sections, in order to have the option of establishing multiple sequential, concurrent or cross-reference comparisons. The evaluation period should cover at least 6 months after the complete "purge" of the string of balls being substituted, which may well take from 3 to 6 months. As a result, the evaluation period required for reliable conclusions should typically exceed 9 to 12 months of fairly undisrupted, normal operation.

The methodologies here proposed make intensive use of **Moly-Cop Tools**, a software package developed by the international Moly-Cop Grinding Systems organization (formerly, ARMCO Worldwide Grinding Systems) with the specific purpose of helping process engineers, characterize and evaluate the operating efficiency of any given grinding circuit, following standardized methodologies and widely accepted evaluation criteria.

INTRODUCTION

Primary and secondary grinding circuit operators worldwide are fully aware of the significant impact of grinding media consumption on the overall cost structure of any particular, full scale grinding facility. The cost associated to such essential consumable mainly depends on two factors : delivered price and durability (quality) of the grinding media. When comparing different operating conditions, arising from eventual changes in ore type or ball supplier (or simply, a trial of new products from the same supplier), there is a well accepted evaluation criterion that accounts for both media price and quality, referred to as the **Effective Grinding Cost** or the **Cost-Effectiveness** of the application. Any given operating condition is considered to be cost-effective when its unit grinding media cost – normally expressed in $/ton ground - is to some extent reduced, with respect to a nominal reference condition :

Grinding Media Cost = **Balls Price** x **Balls Consumption** (1)
($ /ton ground) *($/ton balls)* *(ton balls/ton ground)*

Under this criterion, an alternative, higher-price grinding media product could be cost-effective if its associated consumption rate is sufficiently lower than the reference media, to yield an also lower grinding cost, as dictated by Equation 1.

For the proper application of the above criterion, it is then a basic requirement to maintain continuously updated and representative indicators of the performance of any particular grinding media type being utilized. In each particular case, media price should always be a known, well defined variable; however, it is not so evident to define how media performance (quality) differences amongst alternative product types or operating conditions could be assessed with reasonable accuracy and precision.

In such context, the current publication proposes a functional theoretical framework – based on the *Linear Wear Model*.

REFERENTIAL THEORETICAL FRAMEWORK
GRINDING MEDIA WEAR KINETICS.

The most widely accepted approach to characterize the slow, sustained consumption (wear) kinetics of grinding bodies in rotary tumbling mills is known as the *Linear Wear Theory (1,2)*; according to which - at every instant 't' after the grinding body was thrown to the mill charge - its rate of weight loss will be directly proportional to its surface area exposed to gradual abrasion and/or corrosion wear mechanisms :

$$\Omega_t = d(m)/d(t) = -k_m A_b \qquad (2)$$

where :

- Ω_t = media consumption rate, kg/hr
- m = ball weight, kg; after t hours in the mill
- A_b = surface area of the ball exposed to wear, m^2
- k_m = mass wear rate constant, $kg/hr/m^2$.

Equivalently, taking into account the geometry of the grinding body (sphere or cylinder), Equation 2 converts to :

$$d(d)/d(t) = -2\,k_m / \rho_b = -k_d \qquad (3)$$

where :

- d = size (diameter) of the grinding body, after t hours in the mill charge, mm
- ρ_b = density of the grinding body, gr/cm^3 or ton/m^3
- k_d = linear wear rate constant, mm/hr.

The above first-order differential equation may be easily integrated for the particular and most frequent practical case in which k_d remains constant with time - that is, k_d is not a function of the instantaneous ball diameter *(Linear Kinetics)* - and the mill is continuously recharged with media of a single size d^R (*monorecharge*). In such case :

$$d = d^R - k_d\, t \qquad (4)$$

indicating that the velocity at which the grinding body loses diameter is constant with time. In other words, if a ball loses 1.0 mm in diameter during the first 100 hours in the mill charge, it will also lose the same 1.0 mm in the second 100 hours in the charge, and in the tenth 100 hours, and so on, until the ball is totally consumed or rejected off the mill.

In those much less frequent cases – when k_d is a function of the instantaneous ball diameter (Non-Linear Kinetics) – more complex, but also available (3), models should be applied.

Equation 4 is the basis for the simultaneous, experimental determination of k_d constants of many different types of balls, all present in the any given mill charge at the same time; a methodology referred to as Marked Ball Wear Tests (MBWT) (4).

APPLICATION OF THE LINEAR WEAR MODEL TO FULL SCALE MILLS.

In order to maintain a constant inventory (hold-up) of grinding media in the mill - normally measured by the ratio J_b of the *apparent* volume of balls (i. e., including interstitial spaces in between the balls) to the total effective internal mill volume – operators must continuously compensate for the steel being consumed by recharging new media, preferentially of a single size d^R. In this regard, the typical practice of recharging on a shift-per-shift or day-per-day basis may well be considered *continuous*, for all modeling purposes.

Regardless of the recharge frequency - as a reflection of the linear nature of Equation 4 above - the size distribution of the media in the mill charge should approximate the so-called *uniform* size distribution. For example, consider the case of an operator charging 1000 new balls of 50 mm diameter, every day. By the following day, those balls will have a diameter of say (50 – 0.1) = 49.9 mm and he will be adding 1000 new balls of 50 mm. By the next following day, the first charged balls will have a diameter of (50 – 2*0.1) = 49.8 mm and the balls charged just the day before will have a size of (50 – 0.1) = 49.9 mm. So, after 3 days, not accounting for all the older media, the mill will contain 1000 new 50 mm balls, 1000 almost new 49.9 mm balls and 1000 not so new 49.8 mm balls, assuming k_d remained constant at 0.1 mm/day. If this practice is maintained for a sufficiently large number of days, the operator will find that, in the same period that he recharges another 1000 new balls, he will be losing off the mill charge 1000 old ball *cores* or *nuclei* that reached a small enough critical rejection *(scrap)* size. And after then, the mill will contain a *'string'* of equal number (1000 balls) of every possible size; that is, a *uniform* distribution of sizes that can be mathematically described by the simple expression :

$$f_0(d) = 1 / (d^R - d^S) \quad ; \text{for } d^S < d < d^R \tag{5}$$

where d^S (mm) represents the scrap or rejection size, characteristic of the design and operating conditions of each particular application.

In Equation 5, the density function $f_0(d)$ is such that $f_0(d)\,d(d) = d(d)/(d^R - d^S)$ represents the number fraction of balls in the mill whose size falls in the infinitesimal range [d, d + *d*(d)].

The mass size distribution $F_3(d)$, corresponding to the fraction of the total <u>weight</u> W_b (not number) of balls in the *'string'* with size smaller than 'd', may be determined from the population balance relationship :

$$W_b\, F_3(d) = \int_{d^S}^{d} \rho_b\, (\pi d^3/6)\, N\, f_0(d)\, d(d) \tag{6}$$

The (W_b/N) ratio is derived from the integration of Equation 6 over the whole range of sizes $[d^S, d^R]$ imposing that, by definition, $F_3(d^R) = 1.0$ to obtain :

$$(W_b/N) = (\rho_b\, \pi/24)\, [(d^R)^4 - (d^S)^4]/[(d^R) - (d^S)] \tag{7}$$

Then, upon proper substitution and integration Equation 6 reduces to :

$$F_3(d) = [d^4 - (d^S)^4]/[(d^R)^4 - (d^S)^4] \quad ; \text{for } d^S < d < d^R \tag{8}$$

In the special case, when $d^S \rightarrow 0$, the above equation reduces to :

$$F_3(d) = (d/d^R)^4 \quad ; \text{for } 0 < d < d^R \tag{9}$$

Regarding this simpler case, it is interesting to mention that the well known F. C. Bond *(5)* empirically determined and proposed a value of 3.8 for the exponent in the above equation; fairly close to 4.0, the

theoretically derived value. Further, the relatively high value of the exponent in Equation 9 indicates that most of the weight of balls in the charge is distributed in sizes not much smaller than the original make-up size, d^R.

Given that the wear rate of each grinding body is proportional to its own exposed surface area, the integration of Equation 2 – over the whole range of sizes $[d^S, d^R]$ – demonstrates that the overall grinding media consumption rate $\square_t$ (kg steel/operating hours), corresponding to the ensemble of balls in the mill charge, is consequently proportional to the total area exposed A (m^2) by the *'string'* :

$$\Omega_t = -k_m\, A = -\rho_b\, k_d\; A / 2 \tag{10}$$

For a *monorecharge* policy, with balls of size d^R, the total area of the charge may be obtained from the expression :

$$A = \int_{d^S}^{d^R} (\pi d^2)\, N\, f_0(d)\, d(d) \tag{11}$$

which upon substitution of Equations 5 and 7 plus integration yields a simple expression for the so called **Specific Surface Area** of the *'string'* :

$$a = (A/V_{ap}) = 8000\, (1 - f_v)\, [(d^R)^3 - (d^S)^3]/[(d^R)^4 - (d^S)^4] \tag{12}$$

where :

a	=	specific surface area of the charge, m^2/m^3 (apparent)
V_{ap}	=	apparent mill volume occupied by the charge (including interstitial spaces), m^3, calculated as $W_b/\square_b/(1-f_v)$
W_b	=	total weight of balls in the charge, ton
f_v	=	volumetric fraction of interstitial voids; typically 35-40%.

According to this equation, the total surface area exposed by the mill charge is inversely proportional to the make-up ball size. On this basis, smaller balls of same "intrinsic quality" (i. e., same value of k_d) will wear off faster just because of the correspondingly larger total surface area exposed. Therefore, when comparing two alternative operating conditions, any observed difference in Ω_t does <u>not</u> imply the same corresponding difference in media quality if the balls are <u>not</u> of exactly the same size.

In those special cases when two different ball sizes (d_1^R and d_2^R) are continuously charged to the mill, in a proportion $r_1 : r_2$ (by weight), the combined area exposed so generated may be calculated with the expression :

$$a = (A/V_{ap}) = v_1 a_1 + (1 - v_1) a_2 \qquad (13)$$

where :

$$v_1 = r_1 a_2 / [(1 - r_1) a_1 + r_1 a_2]$$

and the specific areas a_1 and a_2 are obtained from Equation 12 above, for $d^R = d_1^R$ and $d^R = d_2^R$, respectively. Equation 13 arises from recognizing that each make-up size generates its own independent *'string'* and that, in order for them to be consumed in the $r_1 : r_2$ (by weight) proportion, their total exposed areas (m^2) must be in the same proportion inside the mill.

Finally, substitution in Equation 10 above yields :

$$\Omega_t = - 4000\, k_d\, [\rho_b\, (1 - f_v)\, V_{ap}]\, [(d^R)^3 - (d^S)^3]/[(d^R)^4 - (d^S)^4] \qquad (14)$$

or equivalently,

$$\Omega_t = - 4000\, k_d\, W_b\, [(d^R)^3 - (d^S)^3]/[(d^R)^4 - (d^S)^4] \qquad (15)$$

On this basis, k_d may be easily <u>back-calculated</u> from actual operating records or estimates of Ω_t, W_b, d^R and d^S.

Again, as $d^S \rightarrow 0$, Equation 15 reduces to the much simpler form
:

$$\Omega_t = - 4000\, k_d\, W_b\, /d^R \qquad (16)$$

which re-emphasizes the fact that smaller balls will produce higher consumption rates (kg/hr) <u>for the same ball quality</u>, when characterized by k_d.

THE SPECIFIC ENERGY AS A CONTROLLING WEAR KINETICS PARAMETER.

By direct analogy to mineral particle breakage kinetics, it appears reasonable to postulate that an even more representative and scaleable

quality indicator than k_d is the **Energy Specific Wear Rate Constant [k_d^E, µm/(kWh/ton)]**, defined through the expression :

$$k_d = k_d^E\, (P_b/W_b) / 1000 \qquad (17)$$

where the power intensity ratio (P_b/W_b) corresponds to the contribution to mill net power draw P_b (kW) of every ton of balls in the charge (W_b) to the total net power draw P_{net} (kW) of the mill. The underlying theoretical claim is that grinding balls will wear faster in a more power intensive environment. In other words, k_d^E is equivalent to k_d, but proportionally corrected by how much power is being absorbed by each ton of balls in the charge. Therefore, it is to be expected that k_d^E should be more insensitive than k_d to variations in mill operating conditions (that may affect P_b and/or W_b) that may, in turns, produce higher or lower media consumption rates (kg/hr), <u>not</u> caused by variations in grinding media quality. **As a practical evaluation criterion, it should then be accepted that the top quality grinding media, in any given application, will be the one that exhibits the lowest value of the Energy Specific Wear Rate Constant k_d^E**, regardless of the mill operating conditions.

Due application of Equation 17 creates the need for a mathematical representation of the total **Net Power Draw** of the mill in terms of its main dimensions and basic operating conditions. And also, how each component of the mill charge (balls, rocks *(if any)* and slurry) contribute to this total net power demand. An expanded version of the simple Hogg and Fuerstenau *(6,7,8)* model serves such purpose well :

$$P_{net} = \eta\, P_{gross} = 0.238\, D^{3.5}\, (L/D)\, N_c\, \rho_{ap}\, (J - 1.065\, J^2)\, \sin\alpha \qquad (18)$$

where :

- P_{gross} = gross power draw of the mill (kW) = P_{net} / □
- η = overall mechanical and electrical transmission efficiency, °/1
- D = effective internal diameter of the mill, ft
- L = effective internal length of the mill, ft
- N_c = rotational mill speed; expressed as a fraction (°/1) of its critical centrifugation speed : $N_{crit} = 76.6/D^{0.5}$
- J = apparent mill filling, °/1 (including balls, rocks (if any), slurry and the interstitial spaces in between the balls and the rocks, with respect to the total effective mill volume)
- α = charge lifting angle (defines the dynamic positioning of the center of gravity of the mill load (the *'kidney'*) with

respect to the vertical direction. Typically, with values in the range of 35° to 45°.

and where ρ_{ap} denotes the apparent density of the charge (ton/m^3), which may be evaluated on the basis of the indicated charge components (balls, rocks and interstitial slurry) *(8)* :

$$\rho_{ap} = \{ (1-f_v)\, \rho_b\, J_b + (1-f_v)\, \rho_m\, (J - J_b) + \rho_p\, J_p\, f_v\, J \} / J \qquad (19)$$

with :

- f_v = volume fraction (°/1) of interstitial voids in between the balls (typically assumed to be 35-40% of the volume apparently occupied by the balls).
- J_b = apparent balls filling (°/1) (including balls and the interstitial voids in between such balls).
- J_p = interstitial slurry filling (°/1), corresponding to the fraction of the available interstitial voids (in between the balls and rocks charge) actually occupied by the slurry of finer particles.

- ρ_m = mineral particle density, ton/m^3.
- ρ_p = slurry density (ton/m3), directly related to the weight % solids of the slurry (fs) by : 1/[(fs/ρm) + (1 - fs)].

Substitution of Equation 19 into Equation 18 allows for the decomposition of the total net power draw of the mill, in terms of the charge components *(8)*. In particular, the contribution by the balls in the charge becomes :

$$P_b = [(1-f_v)\, \rho_b\, J_b / \rho_{ap}\, J] \cdot P_{net} \qquad (20)$$

Similarly, the contribution to the net mill power by the rocks in the charge becomes :

$$P_r = [(1-f_v)\, \rho_m\, (J - J_b) / \rho_{ap}\, J] \cdot P_{net} \qquad (21)$$

and finally, just for completeness, the contribution of the slurry in the charge becomes :

$$P_s = [\rho_p\, J_p\, f_v\, J / \rho_{ap}\, J] \cdot P_{net} \qquad (22)$$

Back to Equations 15 and 17, an additional formula for the **Energy Specific Media Consumption Rate, Ω_E (grs of steel/kWh drawn)**, may now be derived :

$$\Omega_E = 1000\, \Omega_t / P_b \qquad (23)$$

equivalent to :

$$\Omega_E = 4000\, k_d^E\, [(d^R)^3 - (d^S)^3]/[(d^R)^4 - (d^S)^4] \qquad (24)$$

On this basis, k_d^E may be easily back-calculated from actual operating records or estimates of Ω_E, d^R and d^S; recalling that the top quality grinding media - in any given application - will be the one that exhibits the lowest value of the **Energy Specific Wear Rate Constant k_d^E**, regardless of the mill(s) operating conditions.

Again, as $d^S \rightarrow 0$, Equation 24 reduces to :

$$\Omega_E = 4000\, k_d^E / d^R \qquad (25)$$

which re-emphasizes the fact that smaller balls will produce higher consumption rates (gr/kWh) for the same ball quality, when properly characterized by k_d^E.

PURGE PERIOD.

When comparing two different types of grinding media, an accurate evaluation of their relative performance must necessarily discard all data collected during the so-called *'purge'* period of the mill; that is, the lapse of time required for the complete consumption of the last ball charged prior to the beginning of the test with the alternative media or the time required for the complete consumption of the first new ball charged at the beginning of the test with the alternative media, whichever is longer. Such periods may be readily estimated from Equation 4, setting d = 0 :

$$t_{max} = d^R / k_d \qquad (26)$$

If the test balls are of similar quality, the cummulative grinding media consumption during the whole purge period may be estimated from Equation 14 above (for $d^S \rightarrow 0$) :

$$\Omega_t\, t_{max}/1000 = 4\, \rho_b\, (1 - f_v)\, V_{ap} = 4\, W_b \quad \textbf{!!!} \qquad (27)$$

concluding that the purge period is roughly the time required to consume an amount of steel equivalent to 4 times the tons of balls in the mill load. As a result, a quick calculation of the purge period (months), in any specific application, is simply : [$4W_b$ / (average monthly consumption)]. The resulting value could be as long as 6 to 8 months. In practice, it is considered acceptable to purge the mill for the equivalent of only $2W_b$ as, by that time, there should be no more than 10% of the old *'string'* remaining.

ANALYSIS OF FULL SCALE RESULTS

In the analysis of plant scale data, it is not yet normal practice to base comparisons on estimates of wear rate constants like those described in the preceding section. Instead, one or more of the following consumption indicators are used :

- **Consumption by unit of energy consumed, Ω_E (gr/kWh)** (see Equation 24),
- **Consumption by unit of operating time, Ω_t (kg/hr)** (see Equation 15),
- **Consumption by unit of ore ground, Ω_M (gr/ton)** :

As indicated above, starting from available plant data and corresponding the **Media Charge_Linear Wear** spreadsheet of **Moly-Cop Tools** (3) facilitates the evaluation of these indicators plus the estimate of the more intrinsic wear rate parameters, indicative of the actual grinding media performance in the specific application under consideration.

No doubt, the most commonly used indicator – and unfortunately, the less representative of all – is the specific consumption rate Ω_M normally expressed in (gr of steel/ton ground) that may be also calculated from the expression :

$$\underset{(\,gr/ton\,)}{\Omega_M} = \underset{(\,gr/kWh\,)}{\Omega_E} * \underset{(\,kWh/ton\,)}{E} \qquad (28)$$

where *E* - the specific energy consumed (kWh) per each ton of ore ground - depends exclusively on the operating conditions and intrinsic characteristics of the ore, with no relation whatsoever to the quality of the media being used. It is so concluded that any variation in *E* could be mistakenly interpreted as a variation in media quality, if the Ω_M indicator was to be adopted for comparative evaluations. It is likewise concluded that a much better indicator of media quality is Ω_E, properly corrected by media size d^R - as suggested by Equation 24 - to arrive to the **Energy Specific Wear Rate Constant, k_d^E (μm/(kWh/ton))**.

EXAMPLE OF APPLICATION

Consider the hypothetical case of Fair Mining Co. operating two parallel 36'Φx17' SAG lines. Attachment A & B contains operational records for both lines in the period July '02 thru June '03. The standard 5" balls are being supplied by Forge+. However, beginning November '02, management decided to start an evaluation with the alternative supplier, NKOB.

In the interest of evaluating which supplier provides better quality balls, it must be first recognized that there are multiple forms of establishing such comparison. In fact, discarding 4 months (November '02 thru February '03) as *'purge'* period, at least the following options could be selected :

- **Sequential Evaluation**, comparison of historical consumption rates of the same mill (SAG 2), before and after the purge period.

- **Concurrent Evaluation**, comparison of consumption rates of a test mill (SAG 2) against a standard mill (SAG 1), both operating in parallel, for exactly the same time period, once the purge period has been completed.

In the first case, the following **Sequential Evaluation** could be established :

	SAG 2 Pre Purge	SAG 2 Post Purge	Variation %
ORE THROUGHPUT			
ton/hr	1,254	1,410	12.4
ENERGY CONSUMPTION			
kW (net)	12,058	11,691	(3.0)
kWh/ton	9.62	8.29	(13.8)
BALLS CONSUMPTION			
gr/ton	552	501	(9.2)

If the analysis was to be based only on Ω_M (gr/ton), the first conclusion to be drawn would be that the alternative NKOB balls are 9.2% better than the standard Forge+ balls (?). Even more, the process analyst (or the NKOB vendor) could claim that, thanks to the new balls, throughput was increased by 12.4% ! But the analyst should not ignore that there has been a very significant change in *E* (kWh/ton), before and after the purge period. However, the following more detailed comparison may be established, for the same Sequential Evaluation above, using the spreadsheet **Media Charge_Linear Wear** of **Moly-Cop Tools** (3) :

	SAG 2 Pre Purge	SAG 2 Post Purge	Variation %
ORE THROUGHPUT			
ton/hr	1,254	1,410	12.4
ENERGY CONSUMPTION			
kW (net)	12,058	11,691	(3.0)
kWh/ton	9.62	8.29	(13.8)
BALLS CONSUMPTION			
gr/ton	552	501	(9.2)
kg/hr	692	707	2.2
gr/kWh	57.4	60.4	5.2
Sp. Wear Constant, k_d^E	2.80	2.95	5.3

The additional indicators reveal that, in reality, the alternative media would be, not 9.2% better, but 5.3% worse in performance, as compared to the standard media; proving that Ω_M (gr/ton) is not a reliable media quality indicator, because is distorted by changes in *E* (kWh/ton) strictly related to process variables and totally independent of media quality.

	SAG 1 Post Purge	SAG 2 Post Purge	Variation %
ORE THROUGHPUT			
ton/hr	1,299	1,410	8.5
ENERGY CONSUMPTION			
kW (net)	11,791	11,691	(0.8)
kWh/ton	9.08	8.29	(8.7)
BALLS CONSUMPTION			
gr/ton	479	501	4.6
kg/hr	621	707	13.8
gr/kWh	52.7	60.4	14.6
Sp. Wear Constant, k_d^E	2.55	2.95	15.4

A **Concurrent Evaluation**, based on the same data base of Attachment A shows :

that is, much more significant differences against the NKOB balls than those indicated by the previous comparison. Which one to believe : the sequential or the concurrent evaluations ? There is not a clear cut answer to such question. The merits of the concurrent evaluation is that the comparison is based on the performance of parallel lines, operating for the same periods in time, most likely being fed with the same type of ore. However, no two lines in any Concentrator are really identical and in that regard, the sequential evaluation assures that the comparison is not being affected by intrinsic differences in the process equipment.

A third option for comparisons comes up, referred to as **Cross Reference Evaluation**, which can be equally stated as :

- **Cross Reference A** : difference in the consumption rate of the test mill (SAG 2) - before and after the purge period - minus the same difference for the standard mill (SAG 1), normalized with respect to the standard mill wear constant, before the purge period :

$$[(k_d^E)_{SAG2,after} - (k_d^E)_{SAG2,before}] - [(k_d^E)_{SAG1,after} - (k_d^E)_{SAG1,before}]/(k_d^E)_{A,before} \quad (29)$$

- **Cross Reference B** : difference in the consumption rate of both mills (SAG 1 and SAG 2) - before and after the purge period - normalized with respect to the standard mill wear constant, before the purge period :

$$[(k_d^E)_{SAG2,after} - (k_d^E)_{SAG1,after}] - [(k_d^E)_{SAG2,before} - (k_d^E)_{SAG1,before}]/(k_d^E)_{A,before} \quad (30)$$

For the particular example under consideration, such **Cross Reference** (A and B are mathematically equivalent) would be:

Specific Wear Rate Constant, k_d^E

	SAG 1	SAG 2	
Pre Purge	2.68	2.80	
Post Purge	2.55	2.95	
			10.4 %

indicating that while - because of a change in ore properties or process conditions - SAG1 was reducing the wear rate, SAG 2 could not do the same because of the different grinding media being charged. From a different but equivalent perspective, while SAG 2 was performing slightly worse than SAG 1, prior to the purge period, that difference was incremented after the purge period because of the lower media quality. In both cases, the most reliable estimate of media quality difference is 10.4% worse than the standard Forge+ balls in SAG 1 before the purge. **And if the NKOB balls didn't break at all ?** In such case, all of the observed difference in performance would have to be explained by an even larger value of k_d^E associated to the NKOB balls :

Specific Wear Rate Constant, k_d^E

	SAG 1	SAG 2	
Pre Purge	2.68	2.80	
Post Purge	2.55	3.11	
			16.4%

Or if the NKOB balls were slightly larger, say 130 mm ? In such case, going back to the assumption of similar DBT_{std} performance, the observed difference in performance would also have to be explained by a larger value of k_d^E for the NKOB balls :

Specific Wear Rate Constant, k_d^E

	SAG 1	SAG 2	
Pre Purge	2.68	2.80	
Post Purge	2.55	3.02	
			13.0%

The practice of making balls slightly larger than their nominal size (*'overweight'*) is quite common amongst grinding media manufacturers in their continuous efforts to maximize equipment availability and production capacity. It should have no significant effect on the grinding efficiency of the media but, in terms of wear consumption, it could help hiding 3-5% differences in relative performance, if the consumption indicators are not properly corrected for *'overweight'*.

CONCLUDING REMARKS

The **Linear Wear Model** provides a theoretical framework for the most reliable estimations of comparative grinding media wear performance

(in the absence of breakage) for any given application, on the basis of the **Specific Wear Rate Constant, k_d^E**.

Because of natural process fluctuations and measurement errors, it is not possible to develop a single comparative performance indicator. In such context, evaluations should be ideally conducted in parallel lines, in order to have the option to establish the preferred **Cross Reference** comparisons.

REFERENCES

1. Prentice, T. K., *"Ball Wear in Cylindrical Mills"*, Journal of the Chem., Met. and Mining Society of South Africa, Jan – Feb 1943, pp. 99-131.
2. Norquist, D. E. and Moeller, J. E., *"Relative Wear Rates of Various Diameter Grinding Balls in Production Mills"*, Trans. AIME, Vol. 187, 1950, pp. 712-714.
3. Moly-Cop Tools, Version 1.0, *"Software for the Assessment and Optimization of Grinding Circuit Performance"*, available upon request at molycoptools@molycop.cl
4. Sepúlveda, J. E., *"Experimental Protocol for Marked Ball Wear Tests"*, Internal Report, Moly-Cop Grinding Systems, April 2003, available upon request at jsepulveda@molycop.cl
5. Bond, F. C., *"Crushing and Grinding Calculations"*, Part I, Rev. Jan 2, 1961, Allis Chalmers Co. Publication.
6. Hogg and Fuerstenau, *"Power Relations for Tumbling Mills"*, Trans. SME-AIME, Vol. 252, 1972, pp. 418-432.
7. Gutiérrez, L. and Sepúlveda, J. E., *"Dimensionamiento y Optimización de Plantas Concentradoras Mediante Técnicas de Modelación Matemática"*, CIMM – Chile Publication, 1986.
8. Sepúlveda, J. E., *"A Phenomenological Model of Semiautogenous Grinding Proceses in a Moly-Cop Tools Environment"*, Proceedings SAG 2001 Conference, Vol. 4, pp. 301-315, Vancouver, B. C., Canada.
9. R. Blickenderfer and J. H. Tylczak, *"A Large-Scale Impact Spalling Test"*, WEAR, vol. 84, 1983, pp 361-373.
10. R. Blickensderfer and J. H. Tylczak, *"Laboratory Tests of Spalling, Breaking, and Abrasion of Wear-Resistant Alloys Used in Mining and Mineral Processing"*, USBM RI 8968 [Report], 1985, pp 1-17.

Attachment A
EXAMPLE OF APPLICATION
Fair Mining Co.
Operational Records

Unit :	**SAG 1**	Make-up Balls : 5.0"ϕ	
Mill Diam.	36 ft	% Balls (Nominal)	14
Mill Lenght	17 ft	% Charge (Nominal)	26
% Critical	76 %	% Solids (Nominal)	74
Ore Density	2.8 ton/m3		

	Ore Throughput	Operating hours	Grinding Capacity	Energy Consumption	Mill Power	Balls Consumption				
	ton/month	hr/month	ton/hr	MWh/month	kW	ton/month	gr/ton	kg/hr	gr/kWh	Supplier
Jul '02	1,017,541	721.0	1,411	8,533	11,836	499.21	491	692	58.5	Forge +
Aug	915,593	644.0	1,422	7,639	11,862	375.07	410	582	49.1	Forge +
Sep	908,071	715.0	1,270	8,576	11,994	480.04	529	671	56.0	Forge +
Oct	718,227	643.0	1,117	7,506	11,674	425.99	593	663	56.8	Forge +
Nov	703,180	627.0	1,121	6,960	11,100	358.08	509	571	51.5	Forge +
Dec	852,259	695.0	1,226	7,712	11,096	444.01	521	639	57.6	Forge +
Jan '03	995,836	718.0	1,387	7,872	10,964	513.25	515	715	65.2	Forge +
Feb	1,014,800	691.0	1,469	7,814	11,308	464.15	457	672	59.4	Forge +
Mar	864,302	639.0	1,353	7,606	11,903	400.83	464	627	52.7	Forge +
Apr	935,336	699.0	1,338	8,231	11,775	400.84	429	573	48.7	Forge +
May	867,843	661.0	1,313	8,071	12,210	436.64	503	661	54.1	Forge +
Jun	747,636	631.0	1,185	7,103	11,256	396.00	530	628	55.8	Forge +
Average	878,385	673.7	1,304	7,802	11,581	432.84	493	643	55.5	

Attachment B
EXAMPLE OF APPLICATION
Fair Mining Co.
Operational Records

Unit :	**SAG 2**	Make-up Balls : 5.0"ϕ	
Mill Diam.	36 ft	% Balls (Nominal)	14
Mill Lenght	17 ft	% Charge (Nominal)	26
% Critical	76 %	% Solids (Nominal)	74
Ore Density	2.8 ton/m3		

	Ore Throughput	Operating hours	Grinding Capacity	Energy Consumption	Mill Power	Balls Consumption				
	ton/month	hr/month	ton/hr	MWh/month	kW	ton/month	gr/ton	kg/hr	gr/kWh	Supplier
Jul '02	755,228	632.0	1,195	7,401	11,711	401.29	531	635	54.2	Forge +
Aug	866,067	715.0	1,211	8,314	11,628	483.39	558	676	58.1	Forge +
Sep	845,614	691.0	1,224	8,879	12,849	494.29	585	715	55.7	Forge +
Oct	951,123	688.0	1,382	8,275	12,027	507.00	533	737	61.3	Forge +
Nov	985,943	710.9	1,387	8,629	12,140	370.84	376	522	43.0	NKOB
Dec	701,282	549.0	1,277	6,208	11,308	470.29	671	857	75.8	NKOB
Jan '03	877,346	723.2	1,213	8,459	11,697	431.54	492	597	51.0	NKOB
Feb	916,566	661.1	1,386	7,719	11,675	443.47	484	671	57.5	NKOB
Mar	915,974	678.0	1,351	7,904	11,657	412.35	450	608	52.2	NKOB
Apr	976,000	692.2	1,410	8,329	12,033	457.80	469	661	55.0	NKOB
May	856,863	640.9	1,337	7,296	11,385	511.82	597	799	70.1	NKOB
Jun	1,073,551	700.4	1,533	8,170	11,666	534.28	498	763	65.4	NKOB
Average	893,463	673.5	1,327	7,965	11,828	459.86	515	683	57.7	